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Suggestions for Teachers of Geometry in Grades XI and XII

This pamphlet should be used in connection with Curriculum S.12, Courses of Study in Mathematics, Grades XI and XII.

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THE MINISTER OF EDUCATION

Suggestions Regarding The Revised Course of Study in Geometry

GRADES XI AND XII

I. *General Suggestions:*

1. Where algebraic proofs are employed—and they are valuable in supplementing as well as replacing geometric proofs—care should be taken to maintain the geometric concept.
2. Stress analysis, the use of clear, precise English, the nature of proof, and applications of geometric reasoning to life situations.
3. Divide the exercises, supplementing them where possible and advisable, into three sections:
   (a) arithmetical and algebraic applications
   (b) deductions, as in the present text, but arranged in order of difficulty
   (c) harder deductions for the more able students.
4. Supplement the deductions based on a particular theorem or pair of theorems with sets of exercises of a more comprehensive character, for the solution of which the pupil must keep in mind all the propositions dealt with up to that point.

II. *Comments on the Details of the Course:*

Re Book I: "A Modern Geometry for High Schools"

1. Proposition 10 might well be introduced after proposition 7 and proposition 28 after 16.
2. It should be noted that propositions 30 and 31 are omitted.
3. It is suggested that the topic of "Locus" be introduced at the end of Book I. The discussion should include propositions 16 and 17 of Book II and also that dealing with the locus of points at a given distance from a given straight line.

   The plotting of a course and the mapping of a property from given bearings and distances are interesting applications of this topic and form a useful preliminary to similar work in Trigonometry.
4. The solution of deductions by analysis should be dealt with more fully than is indicated by the text-book treatment. In fact, the method of analysis should continually be employed.

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Re Book II.

1. The proofs of propositions 2 and 7 need not be stressed for examination purposes.

2. The topic of "Projections" and propositions 8, 9, 10 of Book V may well be taken immediately after the theorem of Pythagoras and its converse.

Re Book III:

The proofs for propositions 6, 7, 20, 21 and 24 need not be stressed for examination purposes.

Re Book IV:

1. Propositions 20 to 23 may well be taken immediately after proposition 3.

2. It should be noted that propositions 17 to 19 and 24 are omitted from the regular course of study. However, they might well be drawn to the attention of the more able students.

3. It is important that the topic of "Ratio, Proportion and Elementary Variation" be made an integral part of the work on Book IV and that its algebraic treatment be incorporated in the study of similar triangles. If Algebra has been taken in Grade XI, the work in Ratio, Proportion and Elementary Variation there taken should be reviewed and extended in Grade XII. Certain suggestions regarding the treatment of the topic are given in Section IV.

4. As a practical application of the concept of ratio and introduction to the trigonometry of Grade XIII, the teacher should at this stage deal with the fundamental trigonometric ratios and give the pupils some practice in their use in the solution of right-angled triangles.

III. *Ratio, Proportion and Variation:*

The treatment of this topic will depend on whether the Geometry is taken in Grade XI or Grade XII. In the latter case, it is assumed that a certain amount of work on the topic has been covered as outlined in the course in Algebra. The following outline applies to a Grade XI treatment but may readily be modified for Grade XII. It is essential that the algebraic concepts and the geometrical applications be correlated as intimately as possible.

Definition of ratio, proportion, continued proportion, mean proportional, third proportional.

The following results with related geometric applications:

$$\frac{a}{b}=\frac{ma}{mb}; \quad \text{if} \quad \frac{a}{b}=\frac{c}{d} \quad \text{then } ad = bc, \text{ and}$$

$$\frac{b}{a}=\frac{d}{c}, \quad \frac{a}{c}=\frac{b}{d}; \quad \text{also } a = bK \text{ and } c = dK;$$

$$\text{if} \quad \frac{a}{b}=\frac{c}{d} \quad \text{then} \quad \frac{a}{a+b}=\frac{c}{c+d} \quad \text{etc.}$$

$$\frac{a}{b}=\frac{c}{d}=\frac{e}{f} = \text{etc.} = \frac{a+c+e+}{b+d+f+}$$

Elementary variation—direct, inverse, and simple concrete examples of joint variation. The depth of treatment is suggested in the following section.

IV. *Suggestions Regarding the Treatment of Elementary Variation:*

The topic of "Variation" gives an excellent opportunity for rounding out the work in ratio and proportion, for studying the function of an independent variable, for giving practice in handling formulae, and for illustrating these various aspects of the same basic idea by the graphical method. Further, the mathematical applications are straightforward and practical, and the manipulative work is comparatively simple.

1. In reviewing the theorem "The areas of triangles of equal altitude are proportional to their bases," the pupil is led to see that this fact may be expressed in a different form—"The area of a triangle of constant altitude varies directly as its base." This involves, of course, the discussion of the meaning of the expression "vary directly." The emphasis here, of course, should be on the idea that, as the base increases or decreases in any given ratio, the area of the triangle increases or decreases in the same ratio.

$$\frac{A_2}{A_1}=\frac{b_2}{b_1} \quad \text{if h is a constant}$$

This should be accompanied by numerous illustrations such as: The circumference of a circle varies as the diameter; number of miles travelled at constant speed varies as the time in hours; wages paid a worker varies as the number of hours he is employed; current in a given circuit varies as the applied voltage; pressure of a constant volume of gas varies as the absolute temperature; the area of a circle varies as the square of the radius; the time of swing of a pendulum varies as the square root of its length; etc.

One of the best of these illustrations to use in bringing out the idea of the constant of variation is that involving the circumference of a circle. A discussion leads to the idea that since:

$$\frac{C_1}{C_2} = \frac{D_1}{D_2} \quad \therefore \quad \frac{C_1}{D_1} = \frac{C_2}{D_2} \quad \text{etc.} \quad = \quad K. \quad \text{Hence } C = KD$$

and K is really the number π. Discuss some of the other illustrations similarly.

2. The next step would naturally be to extend the idea to inverse variation, where $x \propto 1/y$. Examples might be:

   The time for a journey varies inversely as the speed; the current obtained from a battery varies inversely as the resistance of the circuit; the volume of a given mass of gas at constant temperature varies inversely as the pressure; the frequency of vibration of a wire varies inversely as its length; the intensity of heat radiation varies inversely as the square of the distance from the source; etc. The class should be assigned a number of numerical examples on each of these phases of the work.

3. We may now go back to the original example and discuss, in an elementary way, the subject of joint variation.

   If we double the base of a triangle of constant altitude, we double its area. If we now treble its altitude, we treble the new area and the final area is six times the original. Hence we gradually develop the idea that $A \propto bh$ or $A = Kbh$. From the familiar formula we recognize that $K = 1/2$.

Use other illustrations such as: Distance travelled depends on time and average speed; wages earned depends on number of hours and rate of pay; volume of cylinder depends on height and area of base; etc. It is important to emphasize the idea that practically all formulae summarize one or more variation concepts.

4. Two important aspects of this topic remain. They may be introduced concurrently with the above but may best, perhaps, be used to review and fix the foregoing ideas.

   If $C \propto D$ or $C = \pi D$, then C is a function of D. D may be considered the independent, and C the dependent, variable. In general, if $y \propto x$, y is a function of x, — about the simplest possible example of the function concept. Further, all these functions may readily be illustrated graphically. Thus the examples

$$C = 2\pi r, \quad A = \pi r^2, \quad V = \frac{4\pi r^3}{3}$$

   might well be plotted on the same pair of axes to portray the varying rates of increase in the dependent variables as r increases.

   In a similar way, it is valuable to study the graph of an example of inverse variation—such as $pv = K$—with or without the introduction of the name "rectangular hyperbola."

5. Finally, a brief reference might be made to such cases of variation as $y = K_1 + K_2 x$ with such simple practical examples as, — the cost of printing programs for a concert consists of two items, one a constant (setting up the type, etc.) and the other varying directly as the number of copies (cost of paper, labour, etc.). Only simple uninvolved examples need be taken.